

SumOfSquares: A Julia package for Polynomial Optimization

Benoît Legat, Tillmann Weisser

November, 2020

History

- Started in August 2016: SOSTOOLS demos.
- October 2016: Amelia Perry, Joey Huchette, Robin Deits and Chris Coey join, package split into MultivariatePolynomials, PolyJuMP and SumOfSquares.
- April 2017, MultivariatePolynomials split into MultivariatePolynomials, DynamicPolynomials, TypedPolynomials,
- August 2017 MultivariateMoments and SemialgebraicSets moved out of SumOfSquares.
- June 2017–March 2019 Development of MathOptInterface (MOI) → rewrite of JuMP, PolyJuMP and SumOfSquares.
- July 2019–December 2020 Development of MutableArithmetics for **performance**.
- 2019–2020 MomentOpt and Tillmann Teweiser push for more **features**.

What is mutable in (JuMP/MOI var: α , Polynomial var: x):

$$A = \begin{bmatrix} \alpha x + 1 & x - \alpha \\ x - \alpha & x + 2 \end{bmatrix}$$

What is mutable in (JuMP/MOI var: α , Polynomial var: x):

$$A = \begin{bmatrix} \alpha x + 1 & x - \alpha \\ x - \alpha & x + 2 \end{bmatrix}$$

3 nested levels of mutability (4 with e.g. BigInt/BigFloat) $\rightarrow$ huge toll on performance.

MutableArithmetics API:

- `MA.operate!` / `MA.operate_to!`: May mutate first argument.
- `MA.mutable_operate!` / `MA.mutable_operate_to!`: Must mutate first argument.

$$p(x) = b(x)^\top Q b(x)$$

Basis b from `MultivariateBases` package:

- `MonomialBasis`: x^2, xy, y^2
- `ScaledMonomialBasis`: $x^2, \sqrt{2}xy, y^2$
- Orthogonal basis: Hermite, Laguerre, Legendre and Chebyshev bases.
- More to come... (e.g. Lagrange interpolation basis)

Ideal certificates (SOS on algebraic variety):

- Remainder: $p(x)$ SOS when $x \in S \rightarrow$ remainder modulo S .
- Newton polytope.
- Maxdegree (all monomials up to degree d).

Preorder certificates (SOS on basic semialgebraic set):

- Putinar certificate:

$$g_i(x) \geq 0 \Rightarrow p(x) \geq 0 \quad \leftrightarrow \quad p(x) = s_0(x) + \sum_i s_i g_i.$$

Flexibility by combining existing or creating new certificates.

$$p(x) = b(x)^\top Q b(x) \rightarrow p(x) = b_1(x)^\top Q_1 b_1(x) + \cdots + b_k(x)^\top Q_k b_k(x)$$

- SignSymmetry certificate.
- Chordal sparsity certificates:
 - VariableSparsity: Variable/correlative sparsity.
 - MonomialSparsity: Monomial/term sparsity.

Chordal *extension* implementation with *greedy* fill-in heuristics. Should move to separate package (with COSMO ?).

Sums of Hermitian squares / non-commutative

$$p(x) = b(x)^\top Q b(x)$$

with $x_1 x_2 \neq x_2 x_1$, $Q \in \mathbb{C}^{n \times n}$ hermitian PSD:

- Initial work on extending MOI to *complex* numbers in `ComplexOptInterface`, e.g.

complex hermitian PSD $\rightarrow$ **real** symmetric PSD.

- *Non-commutative* algebra implemented in `DynamicPolynomials`, i.e. created with `@ncpolyvar`.
- Implementation of Newton polytope for non-commutative variables.
- Still features missing compared to `NCS0Stools`, e.g. atom extraction with `MultivariateMoments`.

Future work

- Polynomial basis for equality (Pull Requests opened by Tillmann Weisser).
- Symmetry reduction (Good progress made by Tillmann Weisser and Marek Kaluba).
- Certificates for PoLyJuMP: equality in ideal without Groebner basis.
- Better integration with Hypatia (just released!)
- More certificates: Schmüdgen, ...
- Rework examples with Literate to include in documentation.